GEOMETRÍA

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SANTILLANA

GEOMETRY

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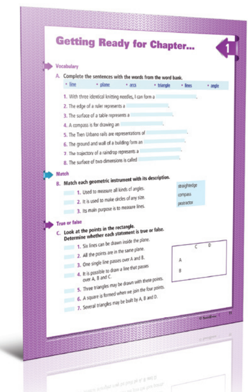
Textbook Structure

Below we describe the types of pages that you will find in each chapter of the textbook *Geometry*. The book consists in 424 pages, distributed in the following manner: eleven chapters, a study guide per chapter, a section with additional problems in section *More Practice on the WEB*, an appendix—containing formulas and symbols that you will be using throughout the chapters—and a visual glossary.



Getting Ready for Chapter...

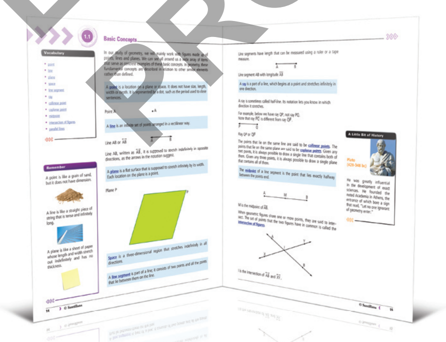
Diagnostic evaluation. It opens each chapter.

The objective of this section is to explore previous knowledge and to review previously acquired concepts and skills that are necessary for the study of the current chapter.

Opening

Each chapter has two opening pages, in which you will find:

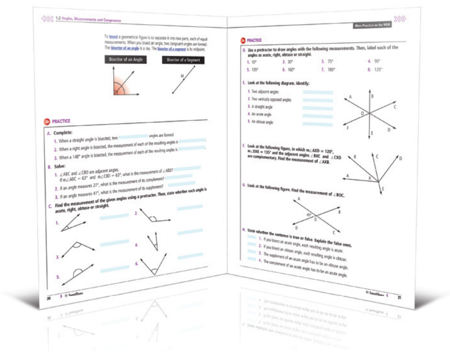
- The chapter's title and number.
- The titles of all the sections that will be studied in the chapter.
- A picture that aims to motivate you to discover the links between real life and mathematical concepts, as well as an opening text that deals with a topic that is related to the chapter's content and is linked to day-to-day life. In addition, it includes questions related to the picture and the opening text, which aim to explore previous experiences and favor oral communication.



Content Pages

Conceptualization pages develop ideas and concepts related to the section's topic. You will see some examples of procedures and techniques that you should learn to solve problems. Some sections are divided into topics to ease understanding.

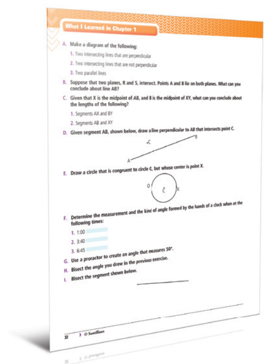
- **Information boxes:** Their purpose is to provide information that is relevant and pertinent to the topic at hand. The boxes are *Remember*, *Take Note*, *Think*, *Did You Know...?* and *A Little Bit of History*.
- ***Your turn***: Placed after some examples throughout the chapter. Its objective is to allow the student to receive feedback from his or her performance, and help the teacher to make any necessary modifications to improve learning achievement.



Practice Pages

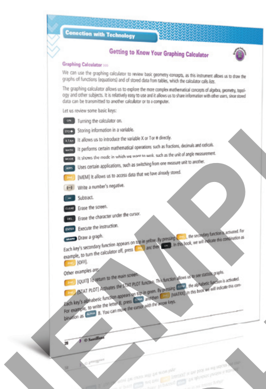
Practice pages propose different activities. One of its objectives is to answer to ability development. These activities will allow you to practice what you have already learned in the previous section.

- **More Practice on the WEB:** The pages on this section present, at the end of each section, a wide variety and number of online exercises to deal with different approaches.



What I Learned in Chapter...

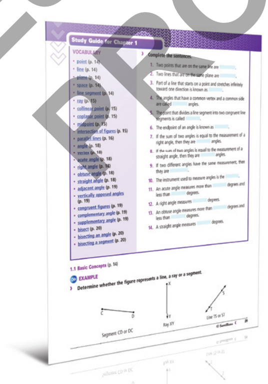
Summative Evaluation. At the end of the chapter, you will see a set of activities and problems that will serve as self-assessment.



Connection with Technology

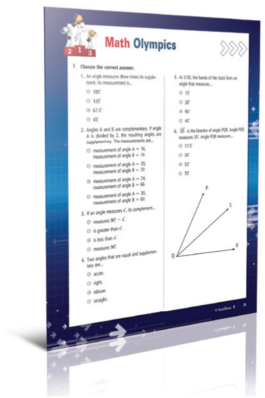
- **Graphing Calculator**
  This section has the purpose of showing how to use a graphing calculator.

- **Text Page**
  This section aims to present the use of the text page.



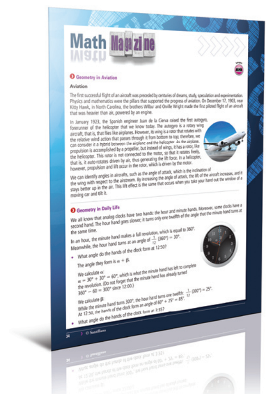
Study Guide for Chapter...

These pages appear at the end of each chapter with a wide variety of activities and problems. Moreover, it includes solved examples with which the important mathematical procedures are clearly explained. These activities will be useful to review all of the chapter's skills.



Math Olympics

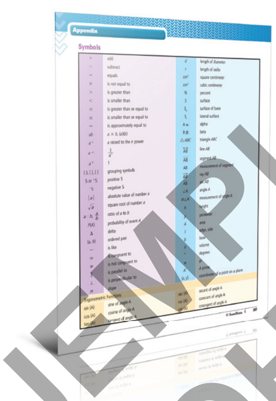
Math competitions to be held in the classroom.



Math Magazine

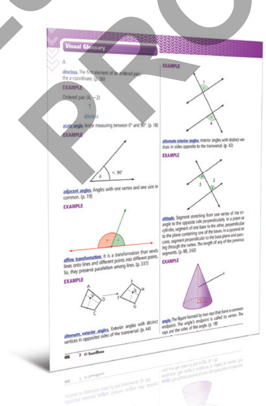
This page contains historic details, entertaining information and curious facts of the world of mathematics. It may contain several sections:

- Dare
- Geometric Project
- Geometry in Daily Life
- Geometry in...
- Trades and Geometry
- In History



Appendix

It collects formulas and symbols that you will use throughout the chapters.



Visual Glossary

Presented at the end of the textbook to ease quick consultation of terms and concepts. The terms are defined, illustrated and ordered alphabetically in English, and are accompanied by the page number where they appear.

Terms are explained in clear and concise language. We have added figures and diagrams because, oftentimes, illustrations make concepts easier and clearer to understand than just definitions.

eBook Structure

Global Locator

This icon tells the student that he or she may access the global locator for an interactive description of where in the world a region or place is located.

Video

This icon indicates that you may watch a video related to an image or topic at hand, which may enrich the chapter's content audiovisually.

Images

This icon will allow you to access more images related to the topic at hand.

Tu escuela **digital**

www.santillanapr.com

This is the first Puerto Rican platform that integrates, in a natural way, the use of TICs (Technology, Computer Sciences and Communications) in harmony with the development of skills, concepts and attitudes.

Santillana's web page (www.santillanapr.com) gives you access to *Tu Escuela Digital*, where you will find resources such as:

- Student's edition of the textbook
- More Practice on the WEB
- Cumulative reviews every two chapters
- Mathematical readings
- Chapter project

Index

Introduction

This book, titled *Geometry*, collects relevant content whose fundamental reason is that it has many different applications in other areas of knowledge: sciences, engineering and, naturally, daily life. The judgment has been writing a book that is complete and extensive, that allows for an approach to content that ensures success, through a very well-thought text and a great number of exercises, both solved and for you to solve.

This is the conductive thread that has guided our writing of this textbook you are holding now. Santillana's textbook *Geometry* introduces the reader into this field of study from the first chapter, through an efficient, clear and practical approach, never forgetting the relationship of this branch of mathematics with other branches, such as algebra.

Geometry is an eminently practical textbook, but it does not ignore the exhaustive and rigorous treatment of the theoretical concepts linked to this field of study. The material's pedagogical approach is based on presenting theory in a direct and accessible way, in order to later reinforce the concepts through practice. All of the mathematical procedures are explained in a clear, precise way, and are also always illustrated with practical examples. This is why Santillana's textbook *Geometry* may be considered a book that *teaches how to do*: to understand fundamental procedures, solve problems and apply knowledge.

Regarding the programming and sequence of content, several criteria have been applied. We have chosen to explain the basic concepts of geometry on the first chapter. On the following chapters, these concepts are reinforced throughout the text, fundamentally the work procedures and the applications to topics of geometry. Another of the book's important characteristics is that the order of the chapters may be modified so as to comply with the needs and goals of the students and teachers. Notwithstanding, we recommend that the chapter sequence is followed as it is.

We hope that the book you are now holding satisfies your expectations and meets the objective that has validated its existence: to contribute with material that really stimulates and eases learning of mathematics.

The Editors

Getting Ready for Chapter... 1

Vocabulary

A. Complete the sentences with the words from the word bank.

• line • plane • arcs • triangle • lines • angle

1. With three identical knitting needles, I can form a ______.
2. The edge of a ruler represents a ______.
3. The surface of a table represents a ______.
4. A compass is for drawing an ______.
5. The Tren Urbano rails are representations of ______.
6. The ground and wall of a building form an ______.
7. The trajectory of a raindrop represents a ______.
8. The surface of two dimensions is called ______.

Match

B. Match each geometric instrument with its description.

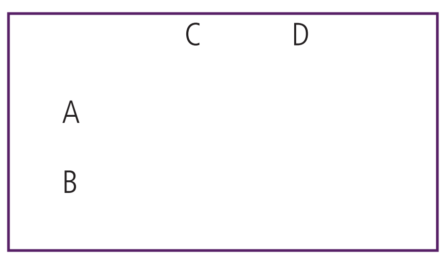
___ 1. Used to measure all kinds of angles.
___ 2. It is used to make circles of any size.
___ 3. Its main purpose is to measure lines.

straightedge
compass
protractor

True or false

C. Look at the points in the rectangle.
Determine whether each statement is true or false.

___ 1. Six lines can be drawn inside the plane.
___ 2. All the points are in the same plane.
___ 3. One single line passes over A and B.
___ 4. It is possible to draw a line that passes over A, B and C.
___ 5. Three triangles may be drawn with these points.
___ 6. A square is formed when we join the four points.
___ 7. Several triangles may be built by A, B and D.



CHAPTER 1

Geometry: Basic Concepts

Contents

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Global Locator | Images | Graphing Calculator

How Did Geometry Originate?

The origin of geometry is linked to the solution of concrete problems, that is, the concepts of geometry are the consequence of practical activities performed by human beings. One of these activities was measuring the Earth; hence the etymology of the word *geometry*: *geo*, "earth" and *metron*, "measurement."

The Greeks built the Parthenon between the years 447 and 432 bc. This temple is located in Athens, and it is the masterpiece of Greek architecture. It is a magnificent marble construction in the doric style. It was erected to honor the Greek goddess Athena. Its approximate dimensions are 69.5 meters long by 30.9 meters wide.

- What geometrical shapes do you identify in the picture?

1.1

Basic Concepts

Vocabulary

- point
- line
- plane
- space
- line segment
- ray
- collinear point
- coplanar point
- midpoint
- intersection of figures
- parallel lines

In our study of geometry, we will mainly work with figures made up of points, lines and planes. We can see all around us a wide array of items that serve as concrete examples of these basic concepts. In geometry, these fundamental concepts are described in relation to other similar elements rather than defined.

A **point** is a location on a plane or space. It does not have size, length, width or depth. It is represented by a dot, such as the period used to close sentences.

Point A • A

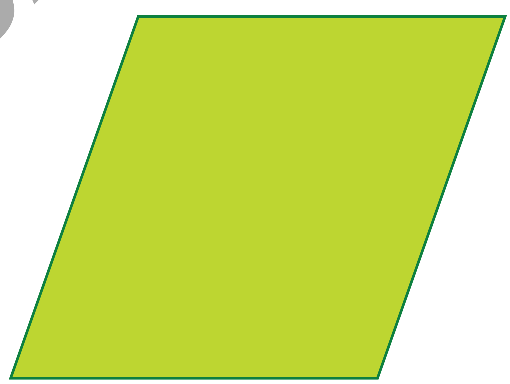
A **line** is an infinite set of points arranged in a rectilinear way.

Line AB or $\overleftrightarrow{AB}$

Line AB, written as $\overleftrightarrow{AB}$, it is supposed to stretch indefinitely in opposite directions, as the arrows in the notation suggest.

A **plane** is a flat surface that is supposed to stretch infinitely by its width. Each location on the plane is a point.

Plane P



P

Space is a three-dimensional region that stretches indefinitely in all directions.

A **line segment** is part of a line; it consists of two points and all the points that lie between them on the line.

Remember

A point is like a grain of sand, but it does not have dimension.

A line is like a straight piece of string that is tense and infinitely long.

A plane is like a sheet of paper whose length and width stretch out indefinitely and has no thickness.

Line segments have length that can be measured using a ruler or a tape measure.

Line segment AB with longitude $\overline{AB}$

A **ray** is a part of a line, which begins at a point and stretches infinitely in one direction.

A ray is sometimes called *half-line*. Its notation lets you know in which direction it stretches.

For example, below we have ray QP, not ray PQ.
Note that ray $\overrightarrow{PQ}$ is different from ray $\overrightarrow{QP}$.

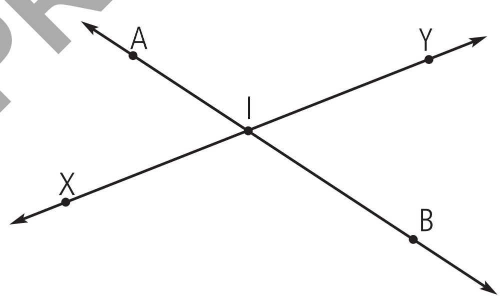
Ray QP or $\overrightarrow{QP}$

The points that lie on the same line are said to be **collinear points**. The points that lie on the same plane are said to be **coplanar points**. Given any two points, it is always possible to draw a single line that contains both of them. Given any three points, it is always possible to draw a single plane that contains all of them.

The **midpoint** of a line segment is the point that lies exactly halfway between the points end.

M is the midpoint of $\overline{AB}$.

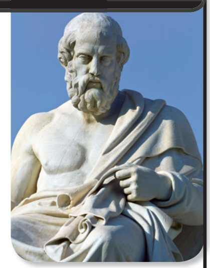
When geometric figures share one or more points, they are said to intersect. The set of points that the two figures have in common is called the **intersection of figures**.



I is the intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{XY}$.

A Little Bit of History

Plato
(420-348 bc)

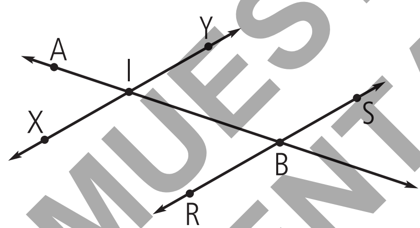


He was greatly influential in the development of exact sciences. He founded the noted Academia in Athens, the entrance of which bore a sign that read, "Let no one ignorant of geometry enter."

Parallel lines are lines that lie on the same plane and never intersect.

EXAMPLE 1

In the figure below, points A, I and B are collinear. So are points X, I and Y. Points A, B, I, X and Y are coplanar. Point I is the intersection of lines AB and XY. It is also the intersection of segment AB and ray XY. Line XY is parallel to line RS.



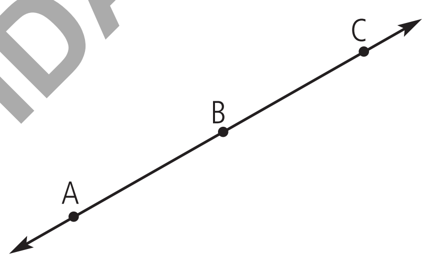
Your turn

Identify a point, a ray, a segment, a line and other collinear points.

PRACTICE

A. Look at the figure and identify a point, a segment, a ray and a line.

1. point
2. segment
3. ray
4. line

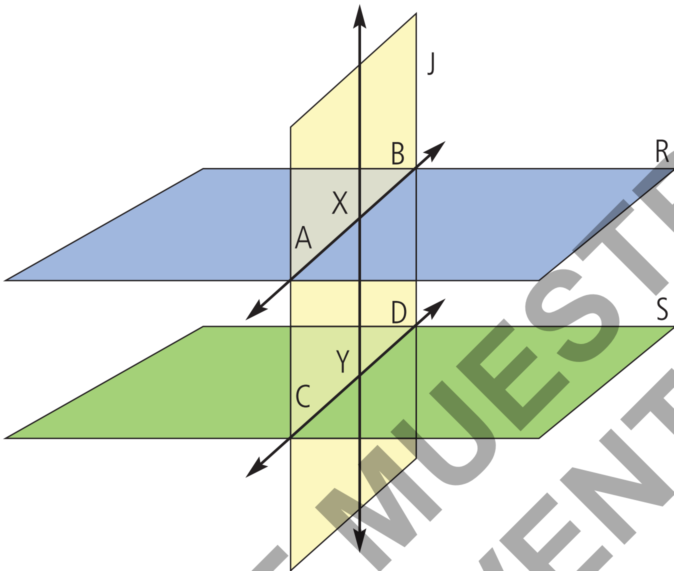


B. Solve:

1. If line segment AB measures 4.8 cm and has midpoint M, then $\overline{AM}$ measures _____.
2. If line segment ST has X as midpoint and $\overline{SX}$ measures 2 inches, then $\overline{ST}$ measures _____.
3. If $\overline{AB}$ has a length of 3 units and $\overline{BC}$ has a length of 2 units, then $\overline{AC}$ has a length of _____ units.

PRACTICE

C. Look at the figure. Then, answer:



1. What is the intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{XY}$?
2. What line is parallel to $\overleftrightarrow{CD}$?
3. What plane is parallel to plane R?
4. Which two lines are parallel?
5. What is the intersection of plane S and $\overleftrightarrow{XY}$?

D. Determine whether each statement is true or false. Justify your answer.

1. The length of a ray can be measured.
2. Planes have borders.
3. It is possible to have two lines in space that do not intersect and that are not parallel.
4. The intersection of two rays is always a point.
5. It is possible for a segment and a ray to lie on the same line, but not have any points in common.
6. Given any three points, it is always possible to draw a single line that contains all of them.
7. Given any four points, it is always possible to draw a single line that contains all of them.
8. The intersection of two distinct planes is always a line.

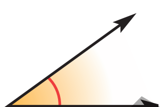
1.2 Angles, Measurement and Congruence

Vocabulary

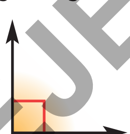
- angle
- vertex
- acute angle
- right angle
- obtuse angle
- straight angle
- adjacent angle
- vertically opposed angle
- congruent figure
- complementary angle
- supplementary angle
- bisect
- midpoint
- bisector of an angle
- bisector of a segment

Remember

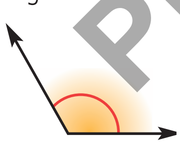
Acute angle: from 0° to 90°



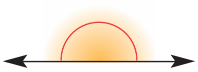
Right angle: 90°



Obtuse angle: from 90° to 180°



Straight angle: 180°

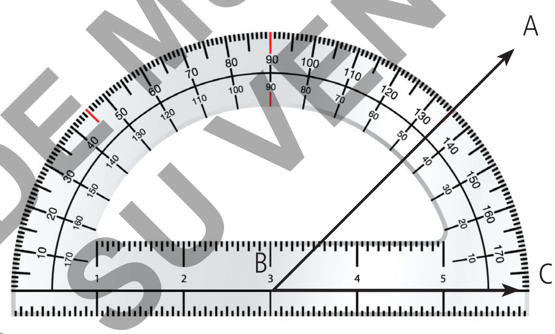


An **angle** is a figure formed by two rays that share a common endpoint. The endpoint of the angle is called its **vertex**. The rays are the angle's sides.

An angle's measure is determined by its opening, not by the length of its sides. A protractor is the instrument used to measure angles. The units we use to measure angles are degrees. Degrees are part of a scale that goes from 0 to 360. A 360° angle indicates a complete turn, while a 180° angle indicates a half-turn.

EXAMPLE 1

Look at the process of measuring a 45° angle with a protractor. We can call it ∠ABC or ∠B. When we represent an angle as ∠ABC, it is supposed that B is its vertex. Since the measurement of ∠ABC is 45°, we will write is as m ∠ABC = 45°.

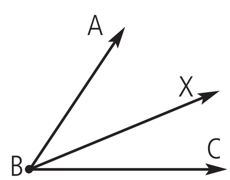


Your turn

Measure a 27° angle and a 97° angle. Then, classify the angles you have drawn.

Angles have their own classification. **Acute angles** measure less than 90°. **Right angles** measure exactly 90°. **Obtuse angles** are greater than 90°, but less then 180°. **Straight angles** measure 180°. You can see, to the left, representations of the different kinds of angles that we have mentioned. Give special notice to the way in which right angles are identified. The symbol ⌉ is used to denote a right angle.

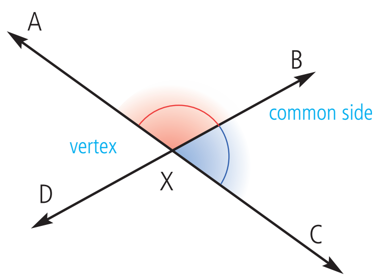
Now that we have classified different angles according to their measurements, let us look at some of their relationships. It is possible to add the measures of angles. If X is a point in the interior of ∠ABC and ray BX is formed, then m∠ABX + m∠XBC = m∠ABC.



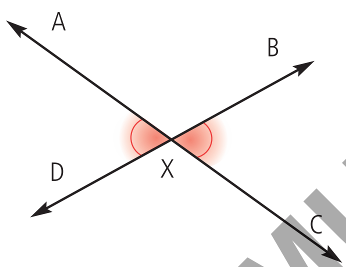
Adjacent angles share a common vertex and one side. When two lines intersect they determine several pairs of angles. **Vertically opposed angles** are a pair of angles that are not adjacent. Perpendicular lines cut themselves forming straight angles.

In the following example, ∠AXB and ∠BXC are adjacent angles. ∠AXD and ∠BXC form one pair of vertically opposed angles.

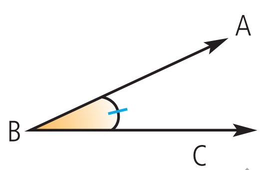
Adjacent Angles

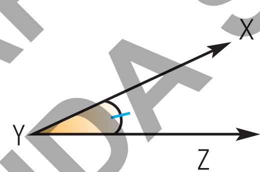


Vertically Opposed Angles



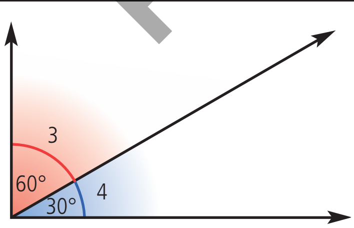
Figures are congruent when they coincide due to superposition. **Congruent figures** have the same size and shape. For example, two congruent angles have the same measurement. If we pick up ∠ABC and lay it on top of ∠XYZ, they would match at each point. Since their measurements are equal, we say that they are congruent and use the following notation: ∠ABC ≅ ∠XYZ.





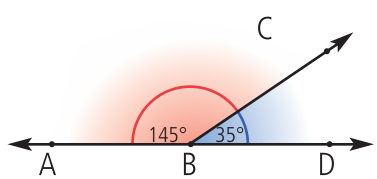
There are two pairs of angles that often arise in geometry tests and problems: complementary angles and supplementary angles. **Complementary angles** are two angles whose measurements sum 90°. When two angles are complementary, we say that one is the complement of the other. **Supplementary angles** are two angles whose measurements sum 180°. When two angles are supplementary, we say that one is the supplement of the other.

Complementary Angles



∠3 is the complement of ∠4.

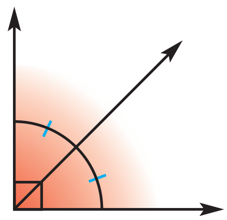
Supplementary Angles



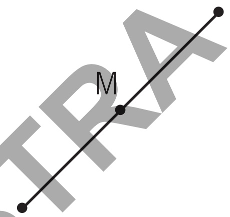
∠ABC is the supplement of ∠CBD.

To **bisect** a geometrical figure is to separate it into two parts, each of equal measurements. When you bisect an angle, two congruent angles are formed. The **bisector of an angle** is a ray. The **bisector of a segment** is its midpoint.

Bisector of an Angle



Bisector of a Segment



PRACTICE

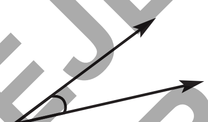
A. Complete:

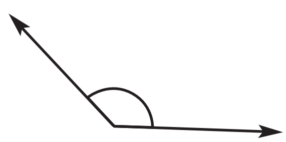
1. When a straight angle is bisected, two ________ angles are formed.
2. When a right angle is bisected, the measurement of each of the resulting angles is ________.
3. When a 148° angle is bisected, the measurement of each of the resulting angles is ________.

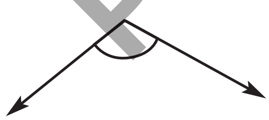
B. Solve:

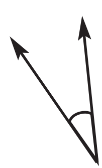
1. ∠ABC and ∠CBD are adjacent angles.
   If $m\angle ABC = 63°$ and $m\angle CBD = 63°$, what is the measurement of ∠ABD? ________
2. If an angle measures 27°, what is the measurement of its complement? ________
3. If an angle measures 41°, what is the measurement of its supplement? ________

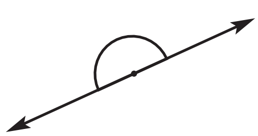
C. Find the measurement of the given angles using a protractor. Then, state whether each angle is acute, right, obtuse or straight.

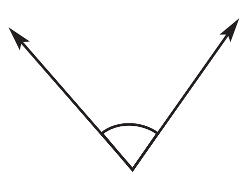
1.  ________

2.  ________

3.  ________

4.  ________

5.  ________

6.  ________

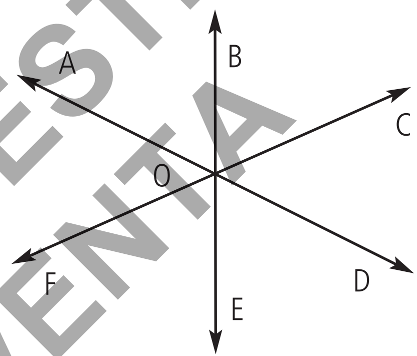
PRACTICE

D. Use a protractor to draw angles with the following measurements. Then, label each of the angles as acute, right, obtuse or straight.

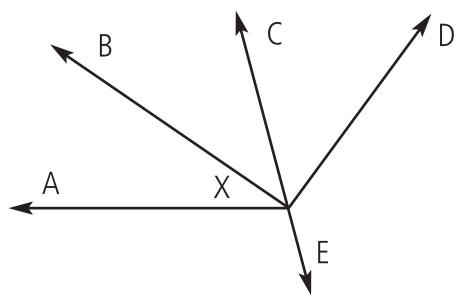
1. 10°
2. 30°
3. 75°
4. 90°
5. 135°
6. 160°
7. 180°
8. 120°

E. Look at the following diagram. Identify:

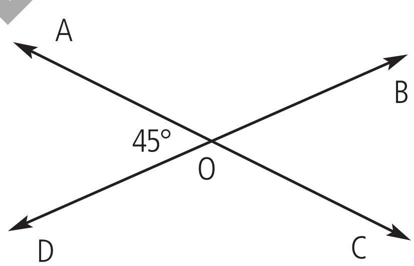
1. Two adjacent angles
2. Two vertically opposed angles
3. A straight angle
4. An acute angle
5. An obtuse angle



F. Look at the following figure, in which m∠AXD = 120°, m∠DXE = 135° and the adjacent angles ∠BXC and ∠CXD are complementary. Find the measurement of ∠AXB.



G. Look at the following figure. Find the measurement of ∠BOC.



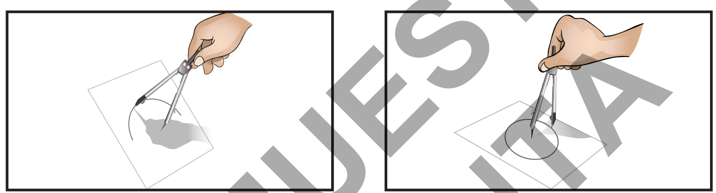
H. State whether the sentence is true or false. Explain the false ones.

1. If you bisect an acute angle, each resulting angle is acute.
2. If you bisect an obtuse angle, each resulting angle is obtuse.
3. The supplement of an acute angle has to be an obtuse angle.
4. The complement of an acute angle has to be an acute angle.

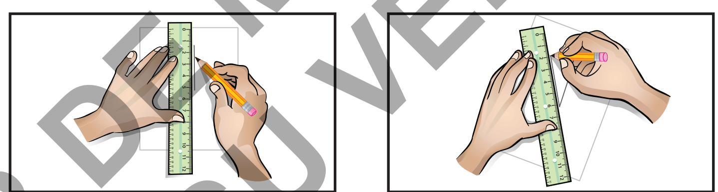
1.3 Geometric Constructions

In this section we will learn how to make accurate drawings of geometric figures. To do that, we will use the same tools that have been used throughout history: a compass and a straightedge. A compass is shown to the left. It has two tips: one is the point and the other is the tip of the pencil. It is not necessary that the straightedge has measurements on it. We can use a ruler, ignoring its marks.

Let's start with the compass. We can easily create either arcs or circles with the compass. The sharp point on the compass serves as a fixed reference and support point. When we use the compass to draw a circle, this point of reference is the center of that circle.



The straightedge can be used to create line segments, rays, lines and angles.

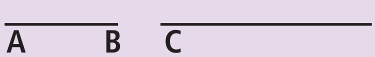


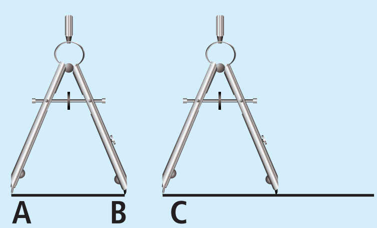
If we are given a segment of a certain length, then we can use both the compass and the straightedge to create a new segment that is congruent to the given segment. For example, given segment AB, follow the steps to create a new segment CD that has the same length.

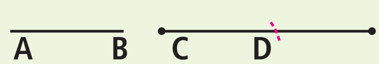
How is it done?

Building a Segment that Is Congruent to a Given Segment

Step 1	Step 2	Step 3
Use the straightedge to draw a line segment that is longer than segment **AB**. Select a point on the new line segment you have drawn and label it **C**.	Take the compass and place the metal point on **A** and the pencil's tip on **B**. Do not change its opening. Place the point of the compass on point **C** and mark an arc on the segment.	Label the point of intersection **D**. As a result, segments **AB** and **CD** are congruent.







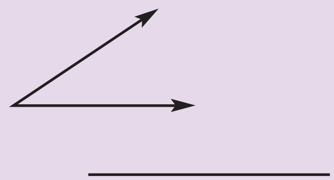
To construct an angle that is congruent to a given angle, follow these steps:

How is it done?

Building an Angle that Is Congruent to a Given Angle

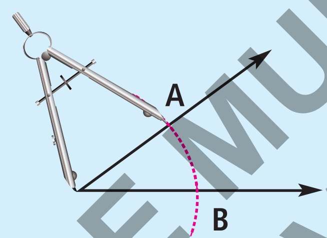
Step 1

Use the straightedge to draw a line segment.



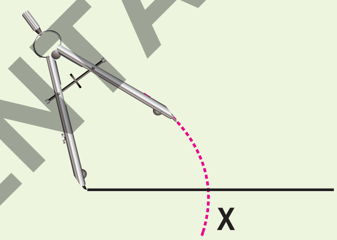
Step 2

Place the point of the compass on the vertex of the given angle and mark an arc. Label this arc **AB**. Do not change the compass's opening.



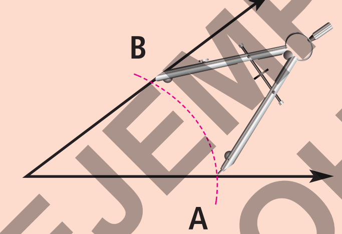
Step 3

Place the point of the compass on one of the endpoints of the line that you drew, and mark an arc. Label **X** the point where the arc intersects the segment.



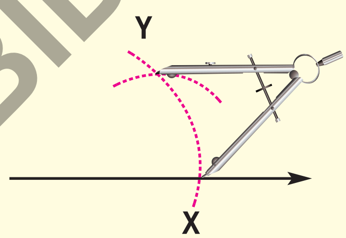
Step 4

Place the point of the compass on point **A** and the tip of the pencil on point **B** of the angle. Do not change the compass's opening.



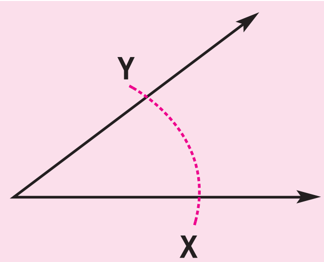
Step 5

Place the point of the compass on point **X** and mark an arc that intersects the previous arc. Label the point of intersection **Y**.



Step 6

Use the straightedge to connect the endpoint of the segment with point **Y**. The angle built will be congruent to the original.



Now that you have seen the compass and straightedge in action, let us go over the basic assumptions applied when we use these tools to construct geometric figures.

First, the distance between the point of the compass and the tip of the pencil remains the same once you set it, unless you reset it.

Second, you can place the point of the compass so that its opening is equal to the distance between two points.

Third, you can place the point of the compass or the straightedge exactly on one point.

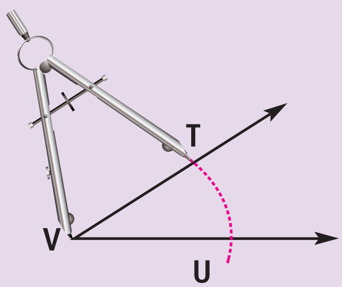
Fourth, given two points, you can always place the straightedge on them so as to draw a line that contains them both.

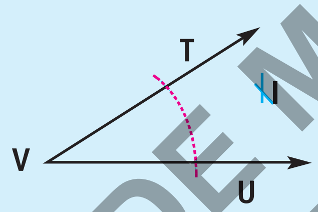
Let us construct another figure. This time we are going to bisect a given angle.

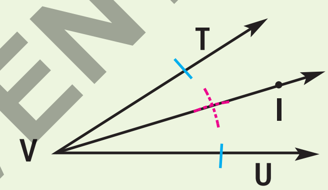
How is it done?

Bisecting an Angle

Step 1	Step 2	Step 3
Place the point of the compass on the vertex of the given angle and draw an arc that intersects both sides. Label the vertex **V** and the two intersection points **T** and **U**.	Place the compass point on point **T** and draw an arc in the interior of ∠**TVU**. Now do the same with **U**. Make sure that the two arcs intersect. Label this intersection point **I**.	Draw a ray from **V** to **I**. Ray **VI** bisects ∠**TVU**.





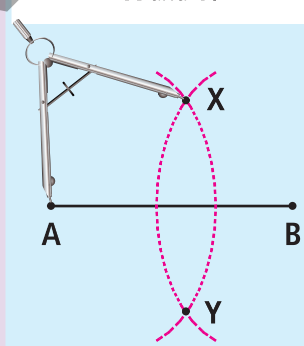


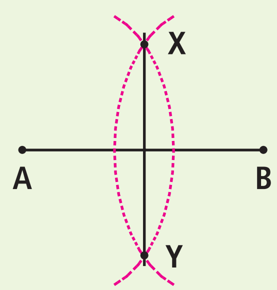
Another very useful construction is finding the perpendicular bisector of a given segment. With this you will obtain a right angle, as well as the midpoint of the given segment.

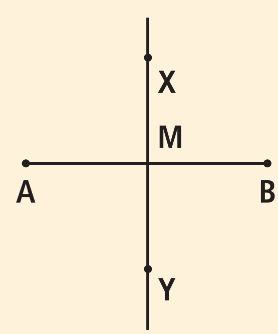
How is it done?

Building the Perpendicular Bisector of a Segment

Step 1	Step 2	Step 3	Step 4
Start with a given segment and label its endpoints **A** and **B**. Set your compass to a distance that is greater than half of segment **AB**.	Place your compass point on point **A** and draw an arc that intersects **AB**. Now do the same on point **B** without resetting your compass. Call those intersection points **X** and **Y**.	Draw a line segment from **X** to **Y**.	Label **M** the point where the new segment intersects **AB**. **M** is the midpoint of segment **AB**. Also, segment **XY** is perpendicular to segment **AB**.







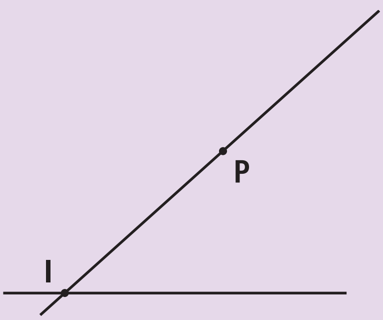
Finally in this section, we will draw a line that is parallel to a line that passes over a given point.

How is it done?

Building a Line that Is Parallel to a Given Line that Passes Over a Given Point

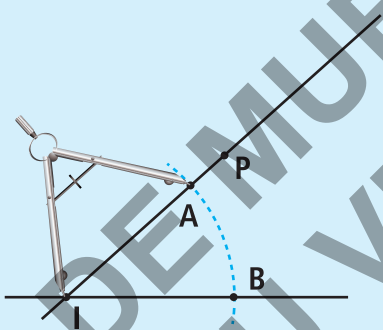
Step 1

Start with a line and a point. Label that point **P**. Use the straightedge to draw a line that passes over **P** and intersects the given line. Label the intersection point **I**.



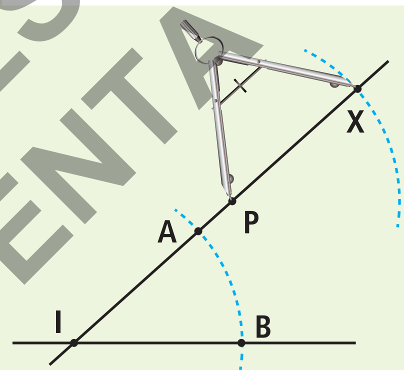
Step 2

Open the compass less than the length of **IP**. Then, place the point of the compass on **I** and draw an arc that intersects **IP**. Label the points of intersection **A** and **B**.



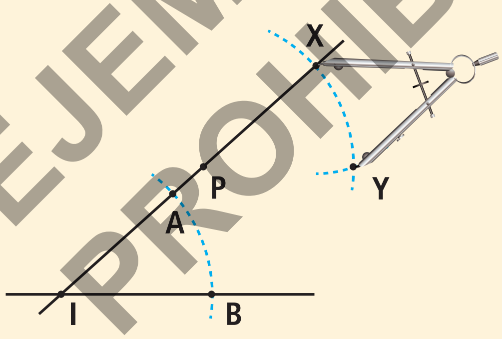
Step 3

Keep your compass setting the same and place its point on **P**. Draw an arc on line **IP** in the opposite direction from **I**, so that it intersects **IP**. Label the intersection point **X**.



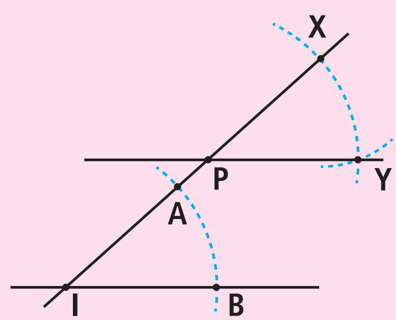
Step 4

Set your compass to the length of **AB**. Then place its point on **X** and mark an arc so that you have an intersection with the arc that goes through **X**. Label the new intersection point **Y**.



Step 5

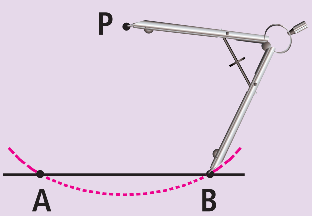
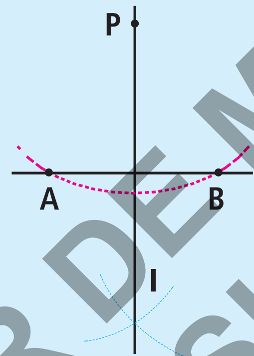
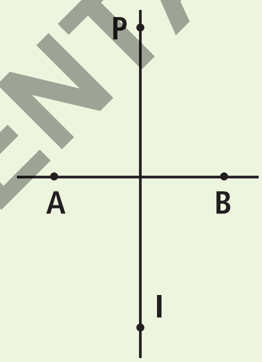
Use the straightedge to draw line **PY**. Line **PY** is parallel to line **IB**.



The following construction consists in finding a line that is perpendicular to a given line that passes over a given point. This construction is similar to the previous one.

How is it done?

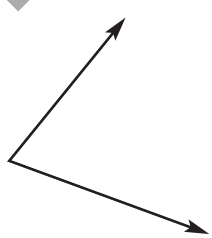
Building a Line that Is Perpendicular to a Line that Passes Over a Given point Outside of Itself

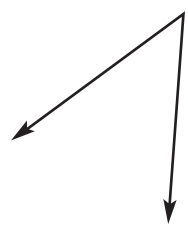
Step 1	Step 2	Step 3
Start with a line and a point. Label that point **P** and place the point of the compass on it. Open the compass so that you can draw an arc that intersects the given line in two points. Label those points **A** and **B**.	Place the point of the compass on point **A** and draw an arc on the opposite side of the line where point **P** is. Do the same for point **B**. Both arcs should intersect. Label this intersection point **I**.	Line **PI** is perpendicular to line **AB**, which passes over point **P**.
		

PRACTICE

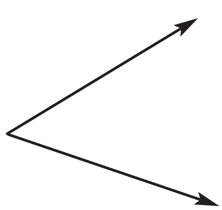
A. Build a congruent figure for each of the following figures:

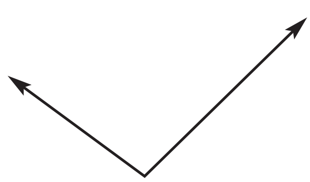
1. 

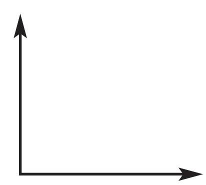
2. 

3. 

B. Bisect the following angles:

1. 

2. 

3. 

PRACTICE

C. **Bisect the given segment.**

D. **Use the compass and the straightedge to build a triangle whose sides are congruent to the three given segments.**

E. **Use the compass and the straightedge to build an equilateral triangle whose sides are congruent to the given segment.**

F. **Use the compass and the straightedge to build the following figures. Do not use the protractor.**

1. An angle whose measurement is 45°
2. A rectangle
3. A line that is perpendicular to the given line and that passes over point X

• X

4. A line that is parallel to the given line and that passes over point X

• X

Getting to Know Your Graphing Calculator

Graphing Calculator

We can use the graphing calculator to review basic geometry concepts, as this instrument allows us to draw the graphs of functions (equations) and of stored data from tables, which the calculator calls *lists*.

The graphing calculator allows us to explore the more complex mathematical concepts of algebra, geometry, topology and other subjects. It is relatively easy to use and it allows us to share information with other users, since stored data can be transmitted to another calculator or to a computer.

Let us review some basic keys:

[ON] Turning the calculator on.

[STO▶] Storing information in a variable.

[X,T,θ,n] It allows us to introduce the variable X or T or θ directly.

[MATH] It performs certain mathematical operations, such as fractions, decimals and radicals.

[MODE] It shows the mode in which we want to work, such as the unit of angle measurement.

[APPS] Uses certain applications, such as switching from one measure unit to another.

[2nd] [MEM] It allows us to access data that we have already stored.

[(−)] Write a number's negative.

[−] Subtract.

[CLEAR] Erase the screen.

[DEL] Erase the character under the cursor.

[ENTER] Execute the instruction.

[GRAPH] Draw a graph.

Each key's secondary function appears on top in yellow. By pressing [2nd], the secondary function is activated. For example, to turn the calculator off, press [2nd] and then [ON]. In this book, we will indicate this combination as [2nd] [OFF].

Other examples are:

[2nd] [QUIT] To return to the main screen.

[2nd] [STAT PLOT] Activates the STAT PLOT function. This function allows us to see statistic graphs.

Each key's alphabetic function appears on top in green. By pressing [ALPHA], the alphabetic function is activated. For example, to write the letter B, press [ALPHA] and then [2nd] [MATRX]. In this book we will indicate this combination as [ALPHA] B. You can move the cursor with the arrow keys.

Study Guide for Chapter 1

VOCABULARY

- point (p. 14)
- line (p. 14)
- plane (p. 14)
- space (p. 14)
- line segment (p. 14)
- ray (p. 15)
- collinear point (p. 15)
- coplanar point (p. 15)
- midpoint (p. 15)
- intersection of figures (p. 15)
- parallel lines (p. 16)
- angle (p. 18)
- vertex (p. 18)
- acute angle (p. 18)
- right angle (p. 18)
- obtuse angle (p. 18)
- straight angle (p. 18)
- adjacent angle (p. 19)
- vertically opposed angles (p. 19)
- congruent figures (p. 19)
- complementary angle (p. 19)
- supplementary angle (p. 19)
- bisect (p. 20)
- bisecting an angle (p. 20)
- bisecting a segment (p. 20)

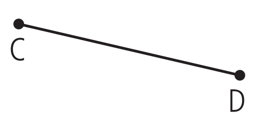
Complete the sentences.

1. Two points that are on the same line are ______.
2. Two lines that are on the same plane are ______.
3. Part of a line that starts on a point and stretches infinitely toward one direction is known as ______.
4. The angles that have a common vertex and a common side are called ______ angles.
5. The point that divides a line segment into two congruent line segments is called ______.
6. The endpoint of an angle is known as ______.
7. If the sum of two angles is equal to the measurement of a right angle, then they are ______ angles.
8. If the sum of two angles is equal to the measurement of a straight angle, then they are ______ angles.
9. If two different angles have the same measurement, then they are ______.
10. The instrument used to measure angles is the ______.
11. An acute angle measures more than ______ degrees and less than ______ degrees.
12. A right angle measures ______ degrees.
13. An obtuse angle measures more than ______ degrees and less than ______ degrees.
14. A straight angle measures ______ degrees.

1.1 Basic Concepts (p. 14)

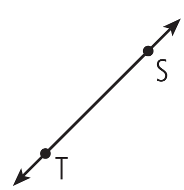
EXAMPLE

Determine whether the figure represents a line, a ray or a segment.



Segment CD or DC

Ray XY



Line TS or ST

PRACTICE

A. **Explain which could be the intersection of a line and a plane: a line, a point or none (that is, that they have no point in common).**

B. **Explain which could be the intersection of two rays: a ray, a point or none (that is, that they have no point in common).**

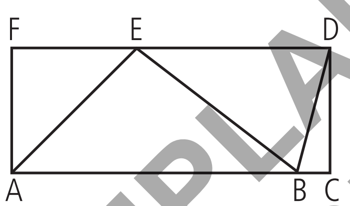
C. **Name the following angles, if the coplanar lines AB and XY intersect in point P.**

1. A pair of adjacent angles ______
2. A pair of vertically opposed angles ______
3. A pair of congruent angles ______
4. A pair of supplementary angles ______

1.2 Angles, Measurement and Congruence (p. 18)

EXAMPLE

› **Look at the figure. Classify the following angles for their measurement: ∠FAE, ∠BCD and ∠AEB.**



Solution: ∠FAE is an acute angle; ∠BCD is a right angle; ∠AEB is an obtuse angle.

PRACTICE

D. **State whether the sentences are true or false. Offer a counterexample to explain the false ones.**

______ 1. The sum of two acute angles is always an acute angle.

______ 2. The sum of two right angles is always a right angle.

______ 3. The sum of two obtuse angles is always an obtuse angle.

______ 4. Two lines in space that never touch are always parallel.

______ 5. A 185° angle is obtuse.

______ 6. The complement of an angle measuring 40° is 140°.

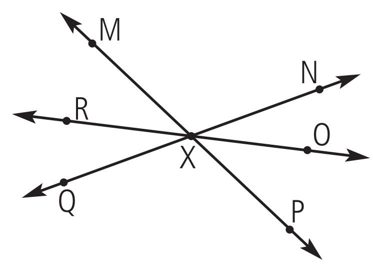
E. **Suppose that M, R, S and T are collinear points. If point M is the midpoint of RS, R is the midpoint of TS and the length of RM is 6 inches, find the length of the following segments. You may make a drawing that helps you interpret the situation.**

1. RS ______
2. TS ______
3. MT ______
4. MS ______
5. RT ______
6. RM ______

F. **Complete:**

Given: $\overleftrightarrow{MP}$, $\overleftrightarrow{RO}$ and $\overleftrightarrow{QN}$ intersect on point X.
m∠RXQ = m∠MXR

Prove that m∠NXO = m∠MXR.



Statement:

1. $\overleftrightarrow{MP}$, $\overleftrightarrow{RO}$ and $\overleftrightarrow{QN}$ intersect on point X.
2. ∠RXQ and ∠NXO are vertically opposed angles.
3. ∠RXQ ≅ ∠NXO
4. m∠RXQ ≅ m∠MXR
5. m∠NXO = m∠MXR

Proof:

1. ______
2. ______

3.

4.

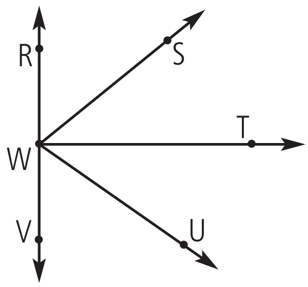
5.

G. **Perform the following proof:**

Given: $\overrightarrow{WT} \perp \overrightarrow{RV}$

$\angle VWU \cong \angle TWS$

Explain why $\angle UWT \cong \angle SWR$.



1.3 Geometric Constructions (p. 22)

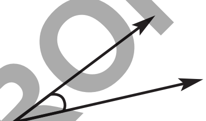
EXAMPLE

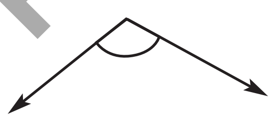
› **Construct a segment that measures twice the measurement of segment HK.**

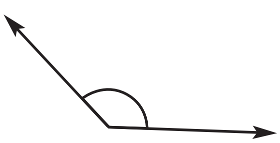
Solution:

PRACTICE

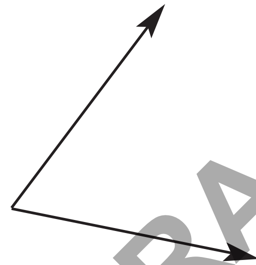
H. **Find the measurement of the following angles using the protractor.**

1. 

2. 

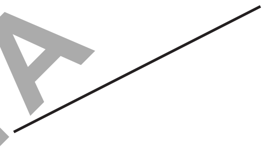
3. 

I. **Bisect the given angle into four smaller congruent angles.**

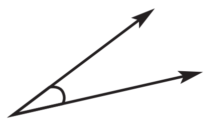


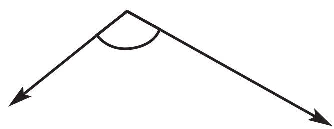
J. **Find the bisector of the following segments using the compass and straightedge.**

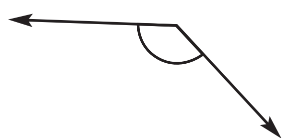
1. 

2. 

K. **Find the bisector of the following angles using the protractor.**

1. 

2. 

3. 

L. **Bisect the given segment into four smaller congruent segments.**

A. **Make a diagram of the following:**

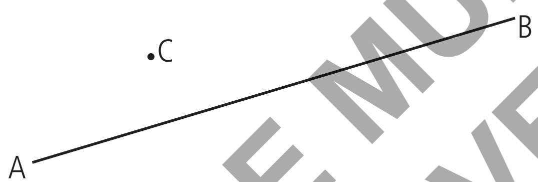
1. Two intersecting lines that are perpendicular
2. Two intersecting lines that are not perpendicular
3. Two parallel lines

B. **Suppose that two planes, R and S, intersect. Points A and B lie on both planes. What can you conclude about line AB?**

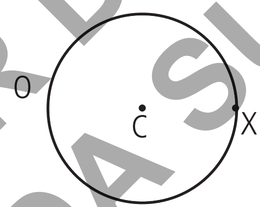
C. **Given that X is the midpoint of AB, and B is the midpoint of XY, what can you conclude about the lengths of the following?**

1. Segments AX and BY
2. Segments AB and XY

D. **Given segment AB, shown below, draw a line perpendicular to AB that intersects point C.**



E. **Draw a circle that is congruent to circle O, but whose center is point X.**



F. **Determine the measurement and the kind of angle formed by the hands of a clock when at the following times:**

1. 1:00
2. 3:40
3. 6:45

G. **Use a protractor to create an angle that measures 50°.**

H. **Bisect the angle you drew in the previous exercise.**

I. **Bisect the segment shown below.**

Math Olympics

Choose the correct answer.

1. An angle measures three times its supplement. Its measurement is...
   - Ⓐ 180°
   - Ⓑ 135°
   - Ⓒ 67.5°
   - Ⓓ 45°

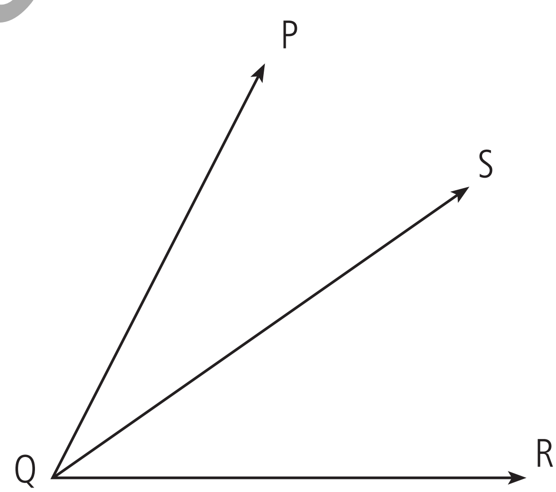
2. Angles A and B are complementary. If angle A is divided by 2, the resulting angles are supplementary. The measurements are...
   - Ⓐ measurement of angle A = 16, measurement of angle B = 74
   - Ⓑ measurement of angle A = 20, measurement of angle B = 70
   - Ⓒ measurement of angle A = 24, measurement of angle B = 66
   - Ⓓ measurement of angle A = 30, measurement of angle B = 60

3. If an angle measures x°, its complement...
   - Ⓐ measures 90° – x°.
   - Ⓑ is greater than x°.
   - Ⓒ is less than x°.
   - Ⓓ measures 90°.

4. Two angles that are equal and supplementary are...
   - Ⓐ acute.
   - Ⓑ right.
   - Ⓒ obtuse.
   - Ⓓ straight.

5. At 3:00, the hands of the clock form an angle that measures...
   - Ⓐ 15°
   - Ⓑ 30°
   - Ⓒ 90°
   - Ⓓ 45°

6. $\overrightarrow{QS}$ is the bisector of angle PQR. Angle PQS measures 35°. Angle PQR measures...
   - Ⓐ 17.5°
   - Ⓑ 35°
   - Ⓒ 55°
   - Ⓓ 70°



Math Magazine

Geometry in Aviation

Aviation

The first successful flight of an aircraft was preceded by centuries of dreams, study, speculation and experimentation. Physics and mathematics were the pillars that supported the progress of aviation. On December 17, 1903, near Kitty Hawk, in North Carolina, the brothers Wilbur and Orville Wright made the first piloted flight of an aircraft that was heavier than air, powered by an engine.

In January 1923, the Spanish engineer Juan de la Cierva raised the first autogyro, forerunner of the helicopter that we know today. The autogyro is a rotary wing aircraft, that is, that flies like airplanes. However, its wing is a rotor that rotates with the relative wind action that passes through it from bottom to top; therefore, we can consider it a hybrid between the airplane and the helicopter. As the airplane, propulsion is accomplished by a propeller, but instead of wings, it has a rotor, like the helicopter. This rotor is not connected to the motor, so that it rotates freely, that is, it auto-rotates driven by air, thus generating the lift force. In a helicopter, however, propulsion and lift occur in the rotor, which is driven by the motor.

We can identify angles in an aircraft, such as the angle of attack, which is the inclination of the wing with respect to the airstream. By increasing the angle of attack, the lift of the aircraft increases, and it stays better up in the air. This lift effect is the same that occurs when you take your hand out the window of a moving car and tilt it.

Geometry in Daily Life

We all know that analog clocks have two hands: the hour and minute hands. Moreover, some clocks have a second hand. The hour hand goes slower; it turns only one twelfth of the angle that the minute hand turns at the same time.

In an hour, the minute hand makes a full revolution, which is equal to 360°. Meanwhile, the hour hand turns at an angle of $\frac{1}{12}(360°) = 30°$.

- What angle do the hands of the clock form at 12:50?

  The angle they form is $\alpha + \beta$.

  We calculate α:
  $\alpha = 30° + 30° = 60°$, which is what the minute hand has left to complete the revolution. (Do not forget that the minute hand has already turned $360° - 60 = 300°$ since 12:00.)

  We calculate β:
  While the minute hand turns 300°, the hour hand turns one twelfth: $\frac{1}{12}(300°) = 25°$.
  At 12:50, the hands of the clock form an angle of $60° + 25° = 85°$.

- What angle do the hands of the clock form at 3:35?